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# $1/m_c$ Expansion of $\Omega_b \rightarrow \Omega_c^{(*)}$ Semileptonic Decay Form Factors

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## Abstract

A simple model of  $\Omega_Q$  and  $\Omega_Q^*$  baryons containing one heavy quark  $Q$  is constructed. Amplitudes are represented by loop graphs with one line for the heavy quark and another for the light degrees of freedom. The latter are modelled as a freely-propagating vector particle interacting nonlocally with the heavy baryon and a free heavy quark. It is argued that the physics of confinement plays an inessential role in determining semileptonic decay form factors. The model has a well-defined heavy-quark expansion which has a form consistent (through order  $1/m_Q$ ) with that determined by QCD through the heavy-quark effective theory. The slope of the Isgur-Wise function is consistent with the Bjorken sum rule bound. The effect of the  $\Omega_Q^* - \Omega_Q$  mass splitting on the first-order form factors is examined.

There is presently a large amount of theoretical interest in properties of baryons with one heavy quark  $Q$ . This is because the spin-flavor symmetries of QCD in the limit  $m_Q \rightarrow \infty$  give rise to relations between certain weak decay form factors and to model-independent absolute normalizations for others at zero recoil [1, 2]. More generally, the heavy-quark effective theory (HQET) may be used to derive relations among the many *a priori* independent corrections to the form factors at any order in the expansion in inverse powers of heavy quark masses. (For a review, see [3].) This information serves to organize and classify the unknown quantities of QCD, but to be of real use it remains necessary to have a model with which to perform computations. Of course, the QCD relations must be checked in any particular model for consistency. One case in which such considerations place constraints on model parameters is described in [4]. The purpose of this paper is to define a model for  $\Omega_Q$  and  $\Omega_Q^{(*)}$  baryons and to show that it is consistent with the form of the heavy-quark expansion in QCD at order  $1/m_Q$  as derived in [5], with no extra constraints on its parameters.

In the heavy-quark limit, the  $\Omega_Q$  and  $\Omega_Q^{(*)}$  are the  $s = 1/2$  and  $s = 3/2$  combinations, respectively, of a heavy quark  $Q$  and light degrees of freedom with  $ss$  quantum numbers and unit spin. The latter may be regarded as a vector particle, hereafter referred to as a diquark. A model for baryon transition amplitudes may then be constructed by joining external baryons to a quark-diquark loop with specified vertices and propagators. (Similar models have been defined for mesons in [6] and for  $\Lambda_Q$  baryons in [7].) One of the main advantages of such a model is that recoil effects are incorporated in a completely relativistic way in the loop graphs. An essential physical effect of soft gluons is to damp out the loop momentum integrals, and this may be accomplished by including damping factors at the baryon-quark-diquark vertices. The model vertices are chosen to be those shown in Fig. 1. The Lorentz structure is determined by heavy-quark symmetry [2]. Standard propagators with constant masses  $m_Q$  and  $m$  are used for heavy quark and diquark, respectively; the propagator for the latter is taken to be purely  $g_{\mu\nu}$  since the diquark is not a gauge field. Possible momentum dependence of the diquark mass is neglected. The form of the damping factors at the vertices together with the use of a constant-mass diquark propagator are the chief assumptions of the model.

In the model, there are no bare propagators for the baryons. The complete set of kinetic and mass terms for  $\Omega_Q$  and  $\Omega_Q^*$  is obtained from the graph shown in Fig. 2 for the proper two-point functions (negative inverse propagators)  $i\Sigma$  and  $-ig_{\mu\nu}\Sigma_*$ , respectively. The physical baryon masses  $M_{(*)}$  are determined by requiring that  $\Sigma_{(*)}(\not{p} = M_{(*)}) = 0$  and  $\Sigma'_{(*)}(\not{p} = M_{(*)}) = 1$ , so that the propagators have a simple pole and unit residue at the physical masses. The physical masses are taken to be real. Analysis of the mass functions shows that their real parts have a single zero which satisfies  $m_Q + m < M_{(*)} < m_Q + \hat{\Lambda}_{(*)}$ . Note that  $m < \hat{\Lambda}_{(*)}$  is required for the existence of a zero.

The imaginary parts of the mass functions are nonvanishing for  $p^2 > (m_Q + m)^2$ .

This means that the baryons of the model are actually above threshold for decay into a free heavy quark and a diquark. This is not unexpected, since the particles in the loops propagate freely. Nowhere have the effects of confinement been introduced. We will see, however, that  $\hat{\Lambda}$  is of order 1500 – 1600 MeV; i.e. the characteristic value of loop momenta which contribute to the mass functions of interest is large compared with the scale  $\Lambda_{QCD}$  of confinement in QCD. Therefore, inclusion of the effects of confinement ought to have only a small effect on the real parts of the mass functions, while slightly modifying their imaginary parts by shifting the threshold to a point above  $M_{(*)}$ . Then it ought to be a reasonable approximation to simply drop the imaginary parts of the mass functions entirely. The same holds for the semileptonic decay form factors; they all have imaginary parts for the same reason as the mass functions do, and only the real parts are retained in the model. It was demonstrated in [6] and [7] that this procedure is consistent with the form of the heavy-quark expansion dictated by QCD in the case of mesons and  $\Lambda_Q$  baryons, respectively, that it preserves the Ward identities, and that it leads to sensible physical predictions.

Heavy-quark symmetries relate properties of  $\Omega$  baryons to those of  $\Omega^*$ . For example, they are degenerate in the heavy-quark limit. This can only be true if their damping factors become the same in this limit, i.e.  $\hat{\Lambda}$  and  $\hat{\Lambda}_*$  approach a common value, say  $\Lambda$ , and  $Z$  and  $Z^*$  approach a common value, say  $A\Lambda$ . The scale  $\Lambda$  characterizes the light degrees of freedom, and can be expected to satisfy  $\Lambda \ll m_Q$ . Everything may then be expanded in power series in  $\Lambda/m_Q$ :

$$\hat{\Lambda}_{(*)} = \Lambda \left\{ 1 + \lambda_{(*)} \frac{\Lambda}{m_Q} + \dots \right\} \quad (1)$$

$$M_{(*)}^2 = m_Q^2 + c\Lambda m_Q \left\{ 1 + d_{(*)} \frac{\Lambda}{m_Q} + \dots \right\} \quad (2)$$

$$Z_{(*)} = A\Lambda \left\{ 1 + B_{(*)} \frac{\Lambda}{m_Q} + \dots \right\} \quad (3)$$

Equation (2) is the most general relation consistent with a lowest-order mass difference between baryon and heavy quark which is common to  $\Omega$  and  $\Omega^*$  and is of order  $\Lambda$ :  $M_{(*)} - m_Q \rightarrow \bar{\Lambda} \equiv c\Lambda/2$ .

In a graph such as Fig. 2 with one diquark propagator, it is possible to choose the loop momentum to be the same as the diquark momentum  $k$ . The damping factors act to suppress contributions of  $k$  much larger than  $\Lambda$ . To facilitate the expansion of the heavy-quark propagator, it is convenient to re-write the light momentum as  $\Lambda k$ . Then the heavy-quark propagator carries momentum  $p = M_{(*)}v + \Lambda k$  with  $v^2 = 1$  and has the expansion

$$\frac{-i(\not{p} + m_Q)}{-p^2 + m_Q^2} = \frac{-i}{\Lambda} \left\{ \frac{\not{p} + 1}{D} + \frac{\Lambda}{m_Q} \left[ F_{(*)} \frac{\not{p} + 1}{D^2} + \frac{\not{k} - c/2}{D} \right] + \dots \right\} \quad (4)$$

where  $F_{(*)} = k^2 - c^2/2 + c d_{(*)}$  and  $D = -2k \cdot v - c$ . The expansion of a vertex factor

is

$$\frac{Z_{(*)}^2}{-\Lambda^2 k^2 + \hat{\Lambda}_{(*)}^2} = A^2 \left\{ \frac{1}{D_k} + \frac{\Lambda}{m_Q} \left[ \frac{2B_{(*)}}{D_k} - \frac{2\lambda_{(*)}}{D_k^2} \right] + \dots \right\}, \quad (5)$$

where  $D_k = -k^2 + 1$ . The propagator for light degrees of freedom is

$$\frac{ig_{\mu\nu}}{-\Lambda^2 k^2 + m^2} = \frac{ig_{\mu\nu}}{\Lambda^2 D_\alpha} \quad (6)$$

where  $D_\alpha = -k^2 + \alpha^2$  with  $\alpha = m/\Lambda$ .

The constants  $c$ ,  $A$ ,  $d_{(*)}$  and  $B_{(*)}$  appearing in (2) and (3) are fixed by consideration of the zero and the slope of the mass function as discussed above. To determine  $c$  and  $A$ , it suffices to compute  $\Sigma_{(*)}$  at zeroth order with  $\not{p} = 1$  using the expansions in (4) and (5). The result is

$$\Sigma = \Sigma_* = \frac{\Lambda}{2} \frac{1}{D_k^2 D_\alpha D}, \quad (7)$$

where we adopt the convention that an overall  $4iA^4 \int d^4k/(2\pi)^4$  is understood in any product of  $1/D$ 's, and the real part is implied. The constant  $c$  enters this expression through  $D$ , and is determined in terms of  $\alpha$  by the on-shell condition

$$0 = \frac{1}{D_k^2 D_\alpha D}. \quad (8)$$

(Note that  $\alpha < 1$  is required for the existence of a zero.) The normalization is fixed by  $\Sigma'_{(*)} = 1$ , where the prime denotes differentiation with respect to  $\not{p}$  and is equivalent at zeroth order with  $(2/\Lambda)\partial/\partial c$ . The zeroth-order constant  $A$  is thus determined in terms of  $\alpha$  by

$$1 = \frac{1}{D_k^2 D_\alpha D^2}, \quad (9)$$

in which  $A$  enters according to the convention mentioned above.

A similar analysis at first order yields

$$c d_{(*)} = \frac{c^2}{2} - 1 + \frac{1}{D_k D_\alpha D^2} + \frac{1}{4D_k^2 D_\alpha} + \frac{4\lambda_{(*)}}{D_k^3 D_\alpha D} \quad (10)$$

$$B_{(*)} = \frac{c}{4} - \frac{F_{(*)}}{2D_k^2 D_\alpha D^3} + \frac{\lambda_{(*)}}{D_k^3 D_\alpha D^2} \quad (11)$$

At order  $1/m_Q$ , the hyperfine interaction between the heavy and light quarks moves the  $\Omega_Q^*$  mass up one unit and the  $\Omega_Q$  mass down two units from their common value in the heavy-quark limit. This is accounted for by writing  $\lambda_* = g + h$  and  $\lambda = g - 2h$ , where the parameter  $g$  contributes to the common mass and  $h$  models the heavy-light hyperfine interaction. Equation (10) shows that the  $\Omega_Q^* - \Omega_Q$  mass difference at order  $1/m_Q$  is proportional to  $h$  alone.<sup>1</sup>

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<sup>1</sup>In the case of mesons, there is an “intrinsic” splitting independent of  $h$  which arises because a  $k^2$  term contributes to the vector meson mass function and not to the pseudoscalar. This effect is absent here, since  $k$  does not appear in the numerator of the diquark propagator.

The main quantities of interest in this paper are the form factors for the semileptonic decay  $\Omega_b \rightarrow \Omega_c^{(*)} \ell \nu$ , defined by

$$\langle \Omega_c | \bar{c} \gamma_\mu b | \Omega_b \rangle = \bar{u}' [F_1 \gamma_\mu + F_2 v_\mu + F_3 v'_\mu] u \quad (12)$$

$$\langle \Omega_c | \bar{c} \gamma_\mu \gamma_5 b | \Omega_b \rangle = \bar{u}' [G_1 \gamma_\mu + G_2 v_\mu + G_3 v'_\mu] \gamma_5 u \quad (13)$$

$$\langle \Omega_c^* | \bar{c} \gamma_\mu b | \Omega_b \rangle = \bar{u}'_\nu [v^\nu (N_1 \gamma_\mu + N_2 v_\mu + N_3 v'_\mu) + g_\mu^\nu N_4] \gamma_5 u \quad (14)$$

$$\langle \Omega_c^* | \bar{c} \gamma_\mu \gamma_5 b | \Omega_b \rangle = \bar{u}'_\nu [v^\nu (K_1 \gamma_\mu + K_2 v_\mu + K_3 v'_\mu) + g_\mu^\nu K_4] u. \quad (15)$$

The notation and conventions of [5] are used in these definitions.

In what follows, we will take the limit  $m_b \rightarrow \infty$  and expand the form factors to order  $1/m_c$ . The general expressions deduced in [5] from QCD using the heavy-quark effective theory are displayed in Tables 1 and 2, in which  $\xi'_i = \xi_i + \varepsilon_c(\tilde{\xi}_i - 3\eta_i)$  and  $\xi''_i = \xi_i + \varepsilon_c(\tilde{\xi}_i + 3\eta_i/2)$ , respectively. The expansion parameter is  $\varepsilon_c \equiv \bar{\Lambda}/m_c$ . The definitions of  $\tilde{\xi}_i, \eta_i$  and  $\kappa_i$  in Tables 1 and 2 differ from those in Ref. [5] by having explicit factors of  $\varepsilon_c$  pulled out to make them universal.

The form factors are computed in the model from the graph shown in Fig. 3, and are expanded using (4), (5) and (6). At zeroth order, consistency with the form shown in Tables 1 and 2 is almost trivial, a consequence of the equality of the vertex factors in the heavy-quark limit and of some standard gamma-matrix algebra. The two zeroth-order universal functions of the model are given by

$$\xi_1 = \frac{1}{D_k^2 D_\alpha D D'}, \quad \xi_2 = 0, \quad (16)$$

where  $D' = -2k \cdot v' - c$ . The Isgur-Wise function  $\xi_1$  is equal to 1 at zero recoil  $v = v'$  by virtue of (9). The structure of the denominator has a simple interpretation:  $D_k^2$  comes from the two equal vertex factors,  $D_\alpha$  is the diquark propagator, and  $D$  and  $D'$  are the  $b$ - and  $c$ -quark propagators, respectively. The Isgur-Wise function is completely determined by the dimensionless ratio  $\alpha$  of the diquark mass to the common damping scale  $\Lambda$  in the heavy-quark limit, and thus depends only on the properties of the light degrees of freedom. In the model,  $\xi_1$  is the same function of  $\alpha$  and  $\omega$  as the single Isgur-Wise function  $\zeta$  determining  $\Lambda_b \rightarrow \Lambda_c$  transitions [7]. However, the numerical value of  $\alpha$  is generally different in the two cases. We remark also that the Isgur-Wise functions  $\xi_i$  and  $\zeta$  for baryons are unrelated to their counterpart  $\xi$  for mesons in the present model [7]. (For a model in which they are related, see [8].)

In contrast to the situation at zeroth order, it is a non-trivial test of any model to verify that the first-order corrections have the form shown in Tables 1 and 2. There are fourteen form factors, so there are *a priori* fourteen possible independent universal functions at order  $1/m_c$ . However, QCD implies that only seven of these are independent, and constrains these seven to contribute in the particular pattern shown in Tables 1 and 2. Of the seven, two are constrained to vanish at zero recoil:  $\tilde{\xi}_1(1) = \eta_1(1) = 0$ . Explicit computation shows that the model is indeed consistent with the form allowed by QCD. The seven universal functions occurring at first order

are found in the model to be

$$\tilde{\xi}_1 = \frac{2\chi_* + \chi}{3}, \quad \tilde{\xi}_2 = \frac{-\xi_1}{3(1+\omega)}, \quad \eta_1 = \frac{2(\chi_* - \chi)}{9}, \quad \eta_2 = \frac{2\xi_1}{9(1+\omega)}, \quad \eta_3 = 0 = \kappa_1 = \kappa_2 \quad (17)$$

where  $\chi_{(*)}$  is given by

$$\frac{c}{2}\chi_{(*)} = \left[2B_{(*)} - \frac{c}{2}\right]\xi_1 + \frac{F_{(*)}}{D_k^2 D_\alpha D^2 D'} - \frac{2\lambda_{(*)}}{D_k^3 D_\alpha D D'}. \quad (18)$$

and vanishes at zero recoil by virtue of (11). In this particular model, there are only two new independent functions at order  $1/m_Q$ :  $\tilde{\xi}_1$ , which is independent of the parameter  $h$  representing the hyperfine interaction between heavy and light quarks, and  $\eta_1$ , which is proportional to  $h$ . Both depend on  $\alpha$  as well. The consistency with QCD is valid for arbitrary values of these parameters, without extra constraints. The model predicts the normalizations  $\tilde{\xi}_2(1) = -1/6$  and  $\eta_2(1) = 1/9$ , independently of the parameter values.

It is possible to make rough estimates of  $m$  and  $\Lambda$ , and hence  $\alpha$ . In [6] and [7], reasonable values of  $m_q = 250$  and  $2m_q = 500$  MeV were found for the nonstrange light quark and scalar diquark masses, respectively. The light quark hyperfine interaction causes  $\Sigma_c$  to be heavier than  $\Lambda_c$  by 170 MeV. Hence, a reasonable value for the nonstrange vector diquark mass is  $2m_q + M_{\Sigma_c} - M_{\Lambda_c} = 670$  MeV. A reasonable value for the strange quark mass was found to be  $m_s = 400$  MeV [9]. For the  $ss$  vector diquark mass, it is then natural to take  $m = 2m_s + (M_{\Sigma_c} - M_{\Lambda_c})(m_q/m_s)^2 = 870$  MeV.

In order to estimate  $\Lambda$ , it is necessary to make use of the mass functions of the model in unexpanded form. These functions are fully determined by the diquark mass and the heavy quark mass. Using the above diquark mass together with the preferred value of  $m_c = 1500$  MeV found in the case of mesons [7], the full model applied to  $\Omega_c$  with mass 2711 MeV [10, 11] yields  $\hat{\Lambda} = 1477$  MeV. It is not yet possible to find  $\hat{\Lambda}_*$  because the mass of  $\Omega_c^*$  is unknown, but it will be somewhat larger than  $\hat{\Lambda}$ . We expect  $\Lambda$  to lie in between, perhaps around 1600 MeV. It is thus reasonable to estimate  $\alpha = 0.5 - 0.6$ . For comparison, Table 3 lists values of the analogous parameters for  $\Sigma_c$  and  $\Lambda_c$ .

Unfortunately, it has not been possible to express the universal functions of the model in terms of elementary functions of  $\omega$ . In what follows, we give the results of numerical computations. The Isgur-Wise function  $\xi_1$  is shown in Fig. 4 for  $\alpha = 0.5$  and  $0.6$ . Its slope at zero recoil is  $-1.02$  and  $-1.18$ , respectively. These are less than the upper bound of  $-1/3$ , valid in the case where  $\xi_2 = 0$ , which was recently derived from a Bjorken sum rule in [12]. In fact, the model's Isgur-Wise function satisfies  $\xi_1'(1) < -3/4$  for all values of  $\alpha = m/\Lambda$  between 0 and 1 [7]. An example of a model which does not satisfy the bound derived in [12] may be found in [13].

The first-order universal functions  $\tilde{\xi}_1 = x + yg$  and  $\eta_1 = zh$  are shown in Fig. 5 for the same values of  $\alpha$ . We see that these functions will be numerically rather

small as long as  $g$  and  $h$  are not too large. In the case of mesons, it was possible to extract values of the analogs of  $g$  and  $h$  from fits to the  $B$ ,  $B^*$ ,  $D$  and  $D^*$  masses [14]. The results were  $g^{\text{meson}} = 0.13$  and  $h^{\text{meson}} = 0.19$ .<sup>2</sup> It is reasonable to suppose that  $g$  and  $h$  for baryons are of the same order, so that the first-order universal functions  $\xi_1$  and  $\eta_1$  which are constrained to vanish at zero recoil remain small across the entire spectrum.

The values of the constants  $c$ ,  $A$ ,  $d_{(*)}$  and  $B_{(*)}$ , defined in (2) and (3), are listed in Table 4. The lowest-order mass difference is  $\bar{\Lambda} \sim 1300$  MeV. This compares with  $\sim 1000$  MeV in the  $\Sigma_c^{(*)}$  system, 790 MeV for  $\Lambda_c$  [7] and 500 MeV for  $D^{(*)}$  mesons [14]. We note that the heavy-quark expansion is formally a power series in  $\bar{\Lambda}/2m_c$  [3], and its convergence properties are best in the case of the lowest-lying mesons. Each set of hadronic states which are degenerate in the heavy-quark limit has a different value of  $\bar{\Lambda}$ , and this increases with the mass of the hadronic state for fixed heavy-quark mass. This is true in any model, because  $\bar{\Lambda} \sim M - m_Q$ . The  $\Omega_c^{(*)}$  baryons have  $\bar{\Lambda}/2m_c \sim 0.4$ , and thus may represent the most massive charmed hadrons for which the heavy-quark expansion makes any sense.

The present model is a specific example of the general physical picture discussed in [15], in which current-induced transitions among heavy-light baryons are described by Bethe-Salpeter bound state wave functions. The most general such model treats the light degrees of freedom as two separate spin-1/2 particles, and represents the amplitudes by two-loop graphs. In the heavy-quark limit, two dynamical approximations to the most general ansatz were discussed in [15]. In one case, in which the orbital and spin degrees of freedom of the light quarks are decoupled, there is a relation between the Isgur-Wise function  $\zeta$  for  $\Lambda_b \rightarrow \Lambda_c$  and the two for  $\Omega_b \rightarrow \Omega_c^{(*)}$ . In the notation of the present paper<sup>3</sup>, this reads  $\zeta = (2 - \omega)\xi_1 + (\omega^2 - 1)\xi_2$ . In contrast, the present model gives  $\zeta = \xi_1$  in the artificial case where the respective values of  $\alpha$  are equal, as discussed above. In the second case discussed in [15], the light quarks move independently and the loop integrals factorize into two pieces. The two Isgur-Wise functions are then related by  $\xi_1 = (\omega + 1)\xi_2$ , again in contrast to the present model where  $\xi_2 = 0$  and  $\xi_1 \neq 0$ .

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<sup>2</sup>The definition of  $g$  in [14] differs from the present one by a minus sign.

<sup>3</sup>The functions  $f$  and  $g$  of [15] are related to the present ones by  $f = 2\xi_2$  and  $g = \xi_1 - (\omega + 1)\xi_2$ .

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## References

- [1] N. Isgur, M.B. Wise: Nucl. Phys. B 348 (1991) 276
- [2] H. Georgi: Nucl. Phys. B 348 (1991) 293
- [3] M. Neubert: SLAC Report No. SLAC-PUB-6263, 1993 (to be published in Phys. Rep.)
- [4] M. Neubert, V. Rieckert: Nucl. Phys. B 382 (1992) 97
- [5] C.G. Boyd, D.E. Brahm: Phys. Lett. B 254 (1991) 468
- [6] B. Holdom, M. Sutherland: Phys. Rev. D 47 (1993) 5067
- [7] B. Holdom, M. Sutherland: University of Toronto Report No. UTPT-93-26, hep-ph/9310216, 1992 (unpublished)
- [8] Nguyen Ai Viet: Syracuse University Report No. SU-4240-561, hep-ph/9311276, 1993 (unpublished)
- [9] B. Holdom, M. Sutherland: University of Toronto Report No. UTPT-93-22, hep-ph/9308313, 1992 (unpublished)
- [10] P.L. Frabetti et al., E687 Coll.: Phys. Lett. B 300 (1993) 190
- [11] H. Albrecht et al., ARGUS Coll.: Phys. Lett. B 288 (1992) 367
- [12] Q.P. Xu: University of Alberta Report No. Thy-26-93, hep-ph/9305349, 1993 (to be published in Phys. Rev. D)
- [13] G.V. Efimov, M.A. Ivanov, N.B. Kulimanova, V.E. Lyubovitskij: Z. Phys. C 54 (1992) 349
- [14] B. Holdom, M. Sutherland: Phys. Lett. B 313 (1993) 447
- [15] F. Hussain, J.G. Körner, M. Krämer, G. Thompson: Z. Phys. C 51 (1991) 321



# TABLES

	$\xi'_1$	$\xi'_2$	$\varepsilon_c \xi_1$	$\varepsilon_c \xi_2$	$\varepsilon_c \eta_3$	$\varepsilon_c \kappa_1$	$\varepsilon_c \kappa_2$
$F_1$	$\frac{-\omega}{3}$	$\frac{\omega^2-1}{3}$	$\frac{-\omega}{6}$	$\frac{\omega^2-1}{2}$	0	$\omega+1$	$\omega(\omega+1)$
$G_1$	$\frac{-\omega}{3}$	$\frac{\omega^2-1}{3}$	$\frac{\omega(1-\omega)}{6(\omega+1)}$	$\frac{(\omega-1)^2}{2}$	0	$\omega-1$	$\omega(\omega-1)$
$F_2$	$\frac{2}{3}$	$\frac{2(1-\omega)}{3}$	$\frac{-1}{3(\omega+1)}$	0	$1-\omega$	$\omega$	1
$G_2$	$\frac{2}{3}$	$\frac{-2(\omega+1)}{3}$	$\frac{1}{3(\omega+1)}$	0	$\omega+1$	$-\omega$	-1
$F_3$	$\frac{2}{3}$	$\frac{2(1-\omega)}{3}$	$\frac{1}{3}$	$1-\omega$	$1-\omega$	$-2-\omega$	$-1-2\omega$
$G_3$	$\frac{-2}{3}$	$\frac{2(\omega+1)}{3}$	$\frac{1-\omega}{3(\omega+1)}$	$\omega-1$	$-\omega-1$	$2-\omega$	$2\omega-1$

Table 1:  $1/m_c$  expansion of  $\Omega_b \rightarrow \Omega_c \ell \nu$  form factors.

	$\xi''_1$	$\xi''_2$	$\varepsilon_c \xi_1$	$\varepsilon_c \xi_2$	$\varepsilon_c \eta_3$	$\varepsilon_c \kappa_1$	$\varepsilon_c \kappa_2$
$N_1$	$\frac{-1}{\sqrt{3}}$	$\frac{\omega-1}{\sqrt{3}}$	$\frac{-1}{2\sqrt{3}}$	0	0	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{3}\omega}{2}$
$K_1$	$\frac{-1}{\sqrt{3}}$	$\frac{\omega+1}{\sqrt{3}}$	$\frac{1-\omega}{2\sqrt{3}(\omega+1)}$	0	0	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{3}\omega}{2}$
$N_2$	0	0	$\frac{1}{\sqrt{3}(\omega+1)}$	0	$\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$
$K_2$	0	0	$\frac{1}{\sqrt{3}(\omega+1)}$	0	$-\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$
$N_3$	0	$\frac{2}{\sqrt{3}}$	0	0	$\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$
$K_3$	0	$\frac{-2}{\sqrt{3}}$	0	0	$\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$
$N_4$	$\frac{-2}{\sqrt{3}}$	0	$\frac{-1}{\sqrt{3}}$	0	0	0	0
$K_4$	$\frac{2}{\sqrt{3}}$	0	$\frac{\omega-1}{\sqrt{3}(\omega+1)}$	0	0	0	0

Table 2:  $1/m_c$  expansion of  $\Omega_b \rightarrow \Omega_c^* \ell \nu$  form factors.

	$M(\text{MeV})$	$m(\text{MeV})$	$\hat{\Lambda}(\text{MeV})$	$\Lambda(\text{MeV})$	$\alpha \equiv m/\Lambda$
$\Omega_c$	2711	870	1477	1600(est.)	$0.5-0.6$
$\Sigma_c$	2453	670	1180	1250(est.)	$0.5-0.6$
$\Lambda_c$	2285	500	1010	1000	$\sim 0.5$

Table 3: Comparison of parameter values for various heavy baryons.

$\alpha$	$c$	$A$	$d_{(*)}$	$B_{(*)}$
0.5	1.56	1.31	$0.36 + 0.76\lambda_{(*)}$	$0.32 + 1.50\lambda_{(*)}$
0.6	1.64	1.16	$0.40 + 0.70\lambda_{(*)}$	$0.32 + 1.80\lambda_{(*)}$

Table 4: Parameters of the heavy-quark expansion for  $\alpha = 0.5$  and  $0.6$ .

## FIGURE CAPTIONS

**FIG. 1:** Model baryon-quark-diquark vertices.

**FIG. 2:** Graph for the complete set of kinetic and mass terms (negative inverse propagators) for  $\Omega_Q^{(*)}$ .

**FIG. 3:** Graph for  $\Omega_b \rightarrow \Omega_c$  semileptonic decay form factors.

**FIG. 4:** Model Isgur-Wise function  $\xi_1$  for  $\alpha = 0.5$  and  $0.6$ .

**FIG. 5:** First-order universal functions  $\tilde{\xi}_1 = x + yg$  and  $\eta_1 = zh$  for  $\alpha = 0.5$  (dotted lines) and  $0.6$  (solid lines).

$$\Omega_Q^{*\nu} \text{---} \text{---} \text{---} \begin{array}{c} \text{---} \text{---} \text{---} \mu \\ \swarrow k \\ \text{---} Q \end{array} = \frac{Z_*^2}{-k^2 + \hat{\Lambda}_*^2} g_{\mu\nu}$$

$$\Omega_Q \overset{Mv}{\dashrightarrow} \text{---} = \frac{Z^2}{-k^2 + \hat{\Lambda}^2} \frac{1}{\sqrt{3}} (\gamma_\mu + v_\mu) \gamma_5$$

Figure 1

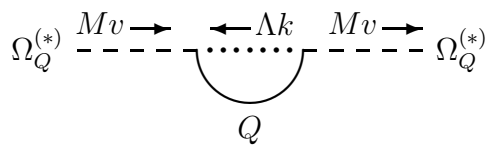


Figure 2

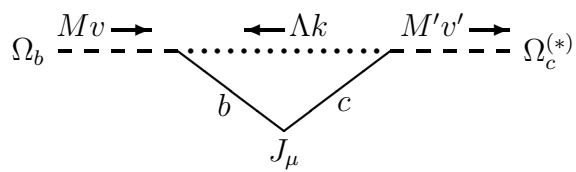


Figure 3

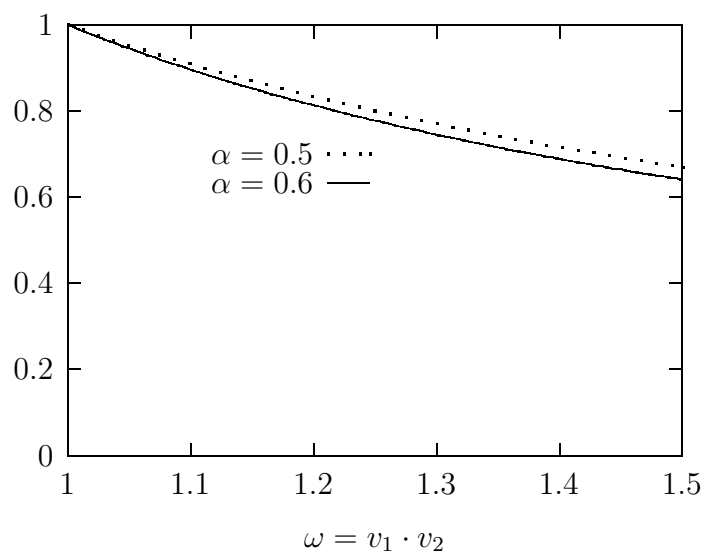


Figure 4

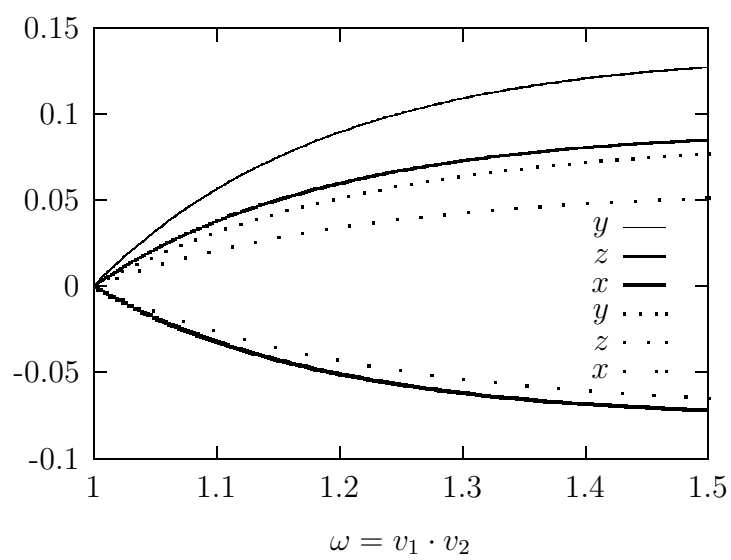


Figure 5